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Origami Instruction in the Middle School Mathematics Classroom: Its Impact on Spatial Visualization and Geometry Knowledge of Students

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Abstract

Within the study of geometry in the middle school curriculum is the natural development of students' spatial visualization, the ability to visualize two- and three-dimensional objects. The national mathematics standards call specifically for the development of such skills through hands-on experiences. A commonly accepted method is through the instruction of Origami, the art of paper folding. This study focused on Origami's impact as a teaching tool in the middle school mathematics classroom. The effects of Origami instruction on a group of seventh grade mathematics students' (n = 56) spatial visualization skills and level of geometry understanding were investigated using a pre-test/post-test quasi-experimental design. A 2 (group) x 2 (gender) factorial method was used on gathered data via three separate spatial tests (Card Rotation, Paper Folding, and Surface Development Tests) and a subset of released National Assessment of Educational Progress questions. After controlling for initial differences, an analysis of covariance revealed a significant interaction effect between group and gender for one of three spatial visualization tests. For geometry knowledge, no significant differences were found. Results imply that Origami lessons blended within mathematics instruction are as beneficial as traditional instruction in building an understanding of geometric terms and concepts, though the approach affects the spatial ability of males and females differently.

Introduction

Geometry is recognized as an important part of the kindergarten through grade 12 mathematics curriculum. It is through geometry that children begin to develop an understanding of "geometric shapes and structures and how to analyze their characteristics and relationships" (National Council of Teachers of Mathematics (NCTM), 2000, p. 41). Part of this development includes the building of spatial visualization skills. Defined as "building and manipulating mental representations of two- and three-dimensional objects and perceiving an object from different perspectives" (p. 41), spatial visualization is viewed as an essential part of geometric thought.

National and international assessments provide the domestic mathematics community with a gauge of student progress. So how are United States youth doing? The third *Trends in International Mathematics and Science Study* (TIMSS) results showed improvement over the years with eighth grade American students performing above the international average. Yet, the results are deceiving with U.S. students scoring as much as 66 points below economic competitors like Japan (National Center for Education Statistics, 2003). One of the areas where students rated weak was geometry. Results from the National Assessment of Educational Progress (NAEP) echoed these findings with eighth grade students showing improvement in geometry over the past decade but still performing well below the proficiency level (Sowder, Wearne, Martin, & Strutchens, 2004). Within the geometry and spatial sense strand, the number of correct problems is low, "… indicating that substantial room for improvement remains in this content area" (p. 124). With younger children, though gains were seen in scores over time, it was felt that they were low considering NCTM's focus on the importance of geometry (Kloosterman et al., 2004).

The *Principles and Standards for School Mathematics* outline specifically what instructional programs should enable children to do when leaving formal schooling (NCTM, 2000). Within geometry, one standard calls for students to "use visualization, spatial reasoning, and geometric modeling to solve problems" (p. 43). This standard focuses on the need to develop students' visualizing skills through active exploration with physical objects and through technology. For middle school-age youth, programs are encouraged to allow students to "examine, build, compose, and decompose complex two- and three-dimensional objects" (p. 237).

The focus on spatial skills as a component of geometry instruction is far from new. A variety of methods are already used by teachers in an attempt to improve students' abilities to visualize and mentally manipulate geometric figures. One such method is Origami, the art of paper folding. A variety of resource books highlight the geometric nature of the art and the abundance of geometric terminology and mathematical concepts inherent in the folding process (Franco, 1999; Gurkewitz & Arnstein, 1995; Pearl, 1994; Tubis & Mills, 2006). Published articles also detail Origami as an instructional tool (Cipoletti & Wilson, 2004; Heukerott, 1988; Wickett, 1996) and describe how to implement the art into the geometry classroom (Geretschlager, 1995; Hall, 1995; Robichaux & Rodrigue, 2003).

Though U.S. students have shown improvement in geometry ability over the past decade, their performance remains lack-luster. NCTM, from their publication of *Curriculum and Evaluation Standards for School Mathematics* (1989) to the recent *Principles and Standards for School Mathematics* (2000), has made a continued attempt to address student performance and outline important content to be covered in the K–12 curricula, emphasizing geometry as one of the five critical areas within mathematics instruction. Of the topics highlighted within geometry, spatial visualization is noted as an important skill.

Seen as a useful tool in the area of spatial visualization, Origami comes up often. NCTM alone has published eight articles on its use as a teaching tool. Of these, many claim Origami has an effect on children's visual skills. For example, in an article on pop-up books, the act of paper folding is said to enable children to "learn three-dimensional geometry concepts" (Huse, Bluemel, & Taylor, 1994, p. 14). Robichaux and Rodrigue (2003) echoed this sentiment focusing on the discussion of Origami in terms of geometry as a way to promote spatial thinking of children. Full books are also dedicated to this process including *Unfolding Mathematics with Unit Origami* (Franco, 1999) and *Math in Motion: Origami in the Classroom* (Pearl, 1994). Both provide a variety of ways that teachers can use the instruction of Origami to help students strengthen their visualizing of two- and three-dimensional figures. Interestingly, though, no research-based evidence exists that Origami indeed has a positive impact on spatial skills. While there have been studies that used a variety of methods aimed at improving skills that included paper folding in some form (Brinkmann, 1966; Dixon, 1995; Tillotson, 1984), no research focused solely on Origami instruction as the teaching method.

The purpose of this research study was to use Origami to improve such spatial abilities. While Origami is commonly discussed and utilized as a tool in the mathematics classroom, little evidence exists to substantiate claims made of its impact on students' understanding of geometry.

The research questions investigated by this study included:

1. Do students who participated in Origami-mathematics lessons integrated into a traditionally instructed geometry unit compared

to students who were instructed solely through traditional instruction show positive gains in ability in terms of (a) spatial visualization skills and (b) geometry knowledge?

2. Do the effects on spatial visualization skills and geometry knowledge of Origami-mathematics lessons differ by gender?

The treatment in this study involved the teaching of Origami-mathematics lessons. These lessons included step-by-step instructions on how to fold an Origami model while interspersing relevant geometric terms and concepts during and after the folding process. The traditional instruction used a textbook-based approach in a geometry chapter from *Holt Middle School Math Course 2* (Bennett et al., 2004).

The theoretical framework of this study lies within Piagetian theory and learning modality research. Within Piagetian theory is the belief that a child possesses three kinds of knowledge from birth: physical, social, and logical-mathematical, the latter being important to this study. Piaget stressed within this area the need for a child to engage in their world to construct knowledge. Piaget and Inhelder (1956) also looked at the development of spatial knowledge. Broken down within stages of exploration and growth of concepts, children are said to reach the “genuine operations” stage around the time they are in middle school. It is at this stage that children have the capacity to fully explore and analyze complex geometric shapes. This supports the use of spatial instruction with middle school youth, as well as the mathematics-enriched and physical nature of the art of Origami.

Learning modalities relates to the process of cognition. Learners have shown preferences for how they receive and synthesize information as noted in the brain-based research literature. Modalities relate directly to the act of instruction; how a teacher instructs content to children. Of those commonly identified by such researchers as Bruner, Olver, and Greenfield (1967) and Samples (1992), are kinesthetic, symbolic abstract, auditory, and visual-spatial. Kinesthetic, as the name suggests, deals with the use of touch and movement to instruct children. Symbolic abstract is the most commonly taught to modality, referring to the traditional forms of practice including writing, reading, and using symbols to represent information such as one would do during an algebra lesson. Auditory learners are those who need to listen and speak about what they are learning. Finally, the visual spatial modality relates to those that need images and spatial experiences to comprehend what they’re learning. By nature, Origami embodies these modalities—combining listening, watching, doing, and seeing with instruction.

A variety of research has sought to improve spatial skills through different specialized training methods. Training came in many forms. Brinkmann (1966) and Ben-Chaim, Lappan, and Houang (1988) each used units integrating a variety of researcher-designed spatial tasks. Doctoral work by Dixon (1995, 1997) and Drickey (2000) had students working on computer-based mathematics programs that allowed students to experiment with two- and three-dimensional objects. Others such as Sundberg (1994) and Battista, Wheatley, and Talsma (1982) used a variety of strategies including manipulation of geometric models and work with traditional mathematics manipulatives such as the geoboard and Mira. Though research differed in method, studies supported the concept that training could improve spatial visualization (Battista et al.; Ben-Chaim et al.; Brinkmann; Dixon, 1995; Sundberg). However, also implied was the recognition that the type of method used to improve spatial visualization would impact to the extent the spatial visualization skills of a student were affected (Dixon, 1997).

Among research reviewed, little is said directly about Origami as a way to target and train spatial abilities of students. In many cases, however, an element of instruction included some form of paper-folding activities (Battista et al., 1982; Brinkmann, 1966; Dixon, 1995; Tillotson, 1984). In one case, a researcher used Origami in a mathematics classroom but was looking for the kind of paper that worked best when instructing blind children (Tinsley, 1972). Though the study did not focus on mathematics content, Tinsley commented that “… the uses of paper-folding in geometry are certainly infinite” (p. 9). Carter and Ferrucci (2002) also supported the use of Origami as a tool in the mathematics classroom while studying the frequency of Origami activities among 10 preservice mathematics teacher texts. With only limited research existing regarding the use of Origami as an instructional tool for improving spatial skills but clear support of its use in the mathematics classroom, this study sought to provide evidence of Origami’s impact on student learning.

Within the design of the study, consideration was made for the gender of the children involved. Traditionally, it is thought that males and females differ in terms of mathematical ability. Though research now exists claiming that differences are negligible at best (Hyde, Fennema, & Lamon, 1990), research continues to show gaps in abilities among males and females in mathematics (Leder, 1990; Voyer, Voyer, & Bryden, 1995). Gender differences also occurred for spatial skills, as Voyer, Voyer, and Bryden found within their meta-analysis. Thus, this factor is of importance when studying children's spatial visualization skills.

Method

Research Design

A quasi-experimental pre-test/post-test design was used in this study. A control group received traditional instruction during a geometry unit in a seventh grade classroom while an experimental group received traditional instruction with a set of interspersed Origami-mathematics lessons. All student participants received 80 minutes of instruction daily over the course of the month-long geometry unit. The experimental group, along with traditional instruction, received Origami instruction three days a week for 20 to 30 minutes at a time, prior to their normal instruction, for the duration of the geometry unit.

Participants

A convenience sample of 56 seventh grade, heterogeneously grouped students who were taking a mathematics class with the same teacher was used for this study. The teacher was selected from a pool of volunteers having the most experience teaching mathematics and a leadership role in the school. With the school's modified block schedule, the teacher had three mathematics classes during the course of a school day. To maintain a balance of males and females and the minimum number needed for statistical analysis, two class sections were combined to create the control group (11 males, 20 females) and the largest remaining section served as the experimental group (14 males, 11 females). All classes were taught by the regular teacher while the experimental group also received Origami lessons from the researcher. Because of the need to accept pre-grouped sets of children, the experimental and control selected were not necessarily equivalent in terms of ability level. To compensate for this, a data analysis method was selected that would remove any differences in ability between the two groups. Differences in languages, ethnicity, and special needs were also considered. In the case of the experimental group, 6 of the 25 students were designated as special needs. Here, an inclusion teacher was present to provide assistance with mathematics presented. The ethnic makeup of two groups was similar and comparable with the school's makeup. Finally, no students were identified as ESL students. Thus, for the most part, beyond ability level, the groups were relatively similar in background.

Instruments

Instruments were selected to measure geometry knowledge and spatial visualization ability. To measure the participants' geometry knowledge, a subset of 27 released National Assessment of Educational Progress (NAEP) multiple-choice questions from the geometry/spatial sense strand written for eighth-grade students was used (National Center for Education Statistics, 2004). Based on a sample of mathematics questions from past NAEP assessments between 1973 and 1996, weighted alpha reliability levels for middle school-age children were reported to be .87 and .85 (Allen, Carlson, & Zelenak, 1999).

With spatial visualization ability involving both two-dimensional and three-dimensional analysis skills, three subtests of the Kit of Factor-Referenced Cognitive Tests (Ekstrom, French, Harman, & Derman, 1976) were selected including the Paper Folding Test, Card Rotation Test, and Surface Development Test. The Card Rotation Test has been reported to measure students' two-dimensional visualization while the Paper Folding Test measures three-dimensional visualizing abilities (Dixon, 1995). The Surface Development Test was used to measure students' ability to move between dimensions (Odell, 1993). Used in both Dixon's and Odell's research, all tests were deemed appropriate for middle school-age children. In addition, reported reliabilities for these spatial tests ranged from .76 to .92 (Fleishman & Dusek, 1971).

Procedure

Prior to the start of the geometry unit, student participants were given pre-tests for geometry knowledge and spatial visualization. To ensure that these middle school students understood how to complete each of these tests, the researcher conducted all testing reviewing instructions for each test prior to its completion. Following this pre-test, the geometry unit began. Each day, both groups received 80 minutes of traditional instruction with their regular classroom teacher (with the inclusive teacher also present with the experimental group).

Over the course of the unit the experimental group received Origami-mathematics lessons, in addition to traditional instruction, taught by the researcher. (See Appendix A for a sample lesson with folding instructions.) These lessons took place three days a week prior to their regular instruction within the 80-minute session. Origami models used ranged from simplistic designs (basic sailboat) to more challenging designs (traditional crane) increasing in difficulty with each visit. A total of twelve Origami-mathematics lessons were integrated over the course of the unit. The regular classroom and inclusion teacher remained present for these lessons but had no involvement during instruction.

Following the completion of the geometry unit, student participants completed the spatial visualization and geometry concept tests. Four weeks had passed from the start of the unit until the final data were collected.

Table 1

Descriptive Statistics for All Instruments

Instrument	**Group**	**Gender (N)**	**Pre-test Mean**	**SD**	**Post-test Mean**	**SD**
Card Rotation Test	Experimental	Male (14)	62.69	17.14	69.00	13.45
		Female (11)	49.27	15.65	48.64	11.87
		Total (25)	56.56	17.46	60.04	16.22
	Control	Male (11)	49.45	14.44	55.82	13.38
		Female (20)	53.85	15.21	62.30	13.37
		Total (31)	52.29	14.85	60.00	13.52
Paper Folding Test	Experimental	Male (14)	4.14	2.25	5.36	1.69
		Female (11)	3.91	2.55	4.55	2.62
		Total (25)	4.04	2.34	5.00	2.14
	Control	Male (11)	3.00	1.18	3.36	2.01
		Female (20)	3.95	1.67	4.85	1.63
		Total (31)	3.61	1.56	4.32	1.89
Surface Development Test	Experimental	Male (14)	10.50	8.30	15.57	9.75
		Female (11)	9.36	5.16	12.64	6.07
		Total (25)	10.00	6.98	14.28	8.30
	Control	Male (11)	5.73	2.83	9.91	6.76
		Female (20)	12.60	7.64	16.00	8.01
		Total (31)	10.16	7.13	13.84	8.04
NAEP Geometry Test	Experimental	Male (14)	14.50	3.80	17.00	3.68
		Female (11)	13.36	5.14	16.09	4.30
		Total (25)	14.00	4.38	16.60	3.91
	Control	Male (11)	15.55	3.70	15.91	4.30
		Female (20)	14.65	4.00	15.55	3.87
		Total (31)	14.97	3.86	15.68	3.96

A recognized threat to the validity of this study was the involvement of the teachers in the study. In an effort to minimize the impact these teachers had, the researcher consulted with both teachers prior to implementation of the treatment. Further, neither teacher was given any prior instruction. They learned the Origami model and participated for the first time with their students. They were told not to discuss this work beyond the experimental group of students as well as not provide any instructional assistance during the lesson beyond restating what was said. These tactics helped to control for potential threats to validity the teachers' involvement might have had.

Analysis

A 2x2 factorial design was used to analyze data. Independent variables of the study included gender and method of instruction. With the research questions focused on Origami's effect on spatial visualization ability and geometry knowledge, these measured skills served as the dependent variables. Due to the need to use a non-random sample, analyses of covariance were used to control for preexisting differences between the control and experimental groups. For each ANCOVA completed, the pre-test score served as the covariate. An ANCOVA was completed for each of three spatial tests and the mathematical achievement test. Within each of these ANCOVAs, gender was used as a fixed factor to detect differences in performance between males and females.

Results

Descriptive statistics for the three spatial visualization tests including the pre- and post-Card Rotation Tests, Paper Folding Tests, and Surface Development Tests as well as the pre- and post-Mathematics Achievement Tests are shown in Table 1. Mean and standard deviation are calculated for each of the instruments.

A 2x2 between-groups ANCOVA was completed on each of the three spatial visualization tests. No statistically significant main or interaction effects were found for either the Paper Folding Test or Surface Development Test (see Tables 3 and 4). However, ANCOVA results from the pre- and post-Card Rotation Test revealed a significant interaction effect between group and gender [$F(1,51) = 9.09$, $p < .005$] with a small effect size (partial eta squared = .15) (see Table 2). To further investigate the result the adjusted means for the Card Rotation Post-Test scores were calculated and represented graphically (see Figure 1). Males who received

Table 2
Analysis of Covariance: Card Rotation Test

Source	SS	*df*	F	*p*
Group	79.52	1	.78	.381
Gender	274.93	1	2.69	.107
Group*Gender	927.33	1	9.09	.004**
Error	5201.31	51		

$^{**}p < .005$

Table 3
Analysis of Covariance: Paper Folding Test

Source	SS	*df*	F	*p*
Group	2.64	1	1.39	.244
Gender	.09	1	.05	.830
Group*Gender	6.82	1	3.59	.064
Error	96.87	51		

Table 4
Analysis of Covariance: Surface Development Test

Source	SS	*df*	F	*p*
Group	2.80	1	.10	.750
Gender	12.37	1	.45	.504
Group*Gender	10.40	1	.38	.540
Error	1393.84	51		

Table 5
Analysis of Covariance: NAEP Geometry Test

Source	SS	*df*	F	*p*
Group	29.65	1	2.96	.091
Gender	.01	1	.00	.977
Group*Gender	.54	1	.05	.817
Error	510.33	51		

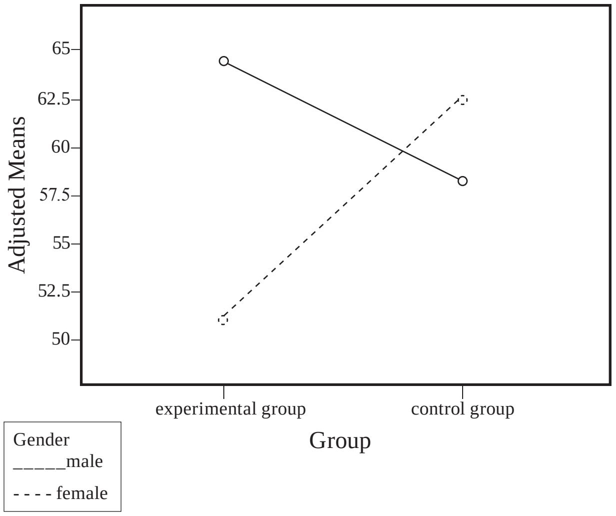

Figure 1. Adjusted Marginal Means of the Card Rotation Post-Test.

Origami instruction along with traditional instruction scored higher than females also receiving Origami instruction. Furthermore, adjusted means of males in the experimental group surpassed male counterparts in the control group whereas females' adjusted mean scores in the experimental group fell below female scores in the control group.

A 2x2 between-groups ANCOVA was also completed for the NAEP geometry test. As shown in Table 5, no statistically significant main or interaction effect was found.

Discussion

In terms of spatial ability, the purpose of this study was to determine how Origami lessons integrated into a traditionally instructed geometry unit would impact students' spatial visualization and overall geometry knowledge. Though previous studies indicate that spatial ability could be improved through specialized training (Battista et al., 1982; Ben-Chaim et al., 1988; Dixon, 1995; Sundberg, 1994), results calculated do not fully support this finding. For both the Paper Folding and Surface Development Tests, no significant differences were found leading to the conclusion that specialized instruction was not more effective than traditional instruction. The Card Rotation Test, however, revealed significant differences based on the group and gender of the participant. This is similar to other studies that show tests involving the mental rotation of images to be most likely to produce gender differences (Casey, Nuttall, & Pezaris, 2001; Voyer et al., 1995).

Of particular interest is the combined impact group and gender had on the Card Rotation Test. Males experiencing Origami instruction seemed to benefit most while their female counterparts actually declined in performance. This is in contrast to research by Ben-Chaim and colleagues (1988), who found that male and female gains in spatial visualization ability were similar after instruction. According to Sanders, Soares, and D'Aquila (1982), this may be a result of males' superior skills in mental rotation. Linn and Petersen (1985) also report that females are more likely to second guess their work, which results in slower performance and lower resultant scores. External factors may also have contributed. Confidence levels vary for males and females in mathematics (Friedman, 1992). This said, girls' lack of confidence like that found in research by Fennema and Sherman (1978), may have impacted their ability to improve their spatial ability through non-traditional means. Boys who are more likely to do more spatially related activities outside of school than girls may be an additional factor that contributed to results found (Casey et al., 2001).

The second aspect of this study was to determine the impact spatial training had on students' geometry knowledge. As in Sundberg's (1994) investigation with a group of middle school students, gains in achievement for treatment and control groups were similar regardless of whether students received specialized or traditional instruction. Using a multiple-choice test containing geometry items much like the one utilized in this study, Sundberg contends that these results may be due to the use of a test that was not carefully matched with the content of the geometry unit. Such may be the case in this study where items were chosen based on their connection to geometry and spatial sense, but not to text content. Test format may also have contributed to findings with students being much more familiar with the multiple-choice standardized test format of the NAEP. This familiarity would reduce anxiety and increase confidence, perhaps dampening potential differences among groups and genders.

Implications

Origami instruction is a widely used practice in the mathematics classroom. Little research to date, though, determines its measurable impact on students. Based on this study, it can be concluded that Origami-mathematics lessons "hold some potential as an instructional aide" (Boakes, 2006, p. 131). In terms of geometry understanding, participants had similar gains in performance, implying that specialized instruction combined with traditional instruction can be as beneficial as traditional textbook-based instruction. Though one is limited within the confines of this study, this finding supports authors' claims of the potential benefits of instructing mathematics through the art of paper folding (Gross, 1992; Hall, 1995; Robichaux & Rodrigue, 2003; Tubis & Mills, 2006; Wickett, 1996).

In terms of spatial visualization, gender and group participation had a combined effect on students' gains in spatial ability for one of the three spatial visualization skills tested. Thus, males and females responded differently to the kind of instruction they were provided. Though research has said that gender differences in spatial abilities have decreased in size over the years (Voyer et al., 1995), they still persist (Ben-Chaim et al., 1988; Casey et al., 2001) as is the case in this study. Future studies seeking to improve spatial visualization through specialized training should take this into account including such factors as choice of spatial test and external factors influencing gender performance.

Recommendations

As in any study, though some insights can be made based on the research conducted, there remains as many questions as answers. For instance, would a different set of spatial tests produce different results? Though the tests were selected based on grade-appropriateness and for the specific spatial visualization skills it assessed, tests not predisposed to gender differences, such as the Card Rotation Test, might have produced varied results.

Participants experienced Origami-mathematics lessons three days a week during the one-month long geometry unit. Limited exposure to such spatial training, according to Casey, Nutall, and Pezaris (2001), is unlikely to be successful. Another question then is if exposure to Origami instruction was extended (not necessarily limited to geometry content), would it have a different effect on performance?

Existing research maintains that although the gender gap in spatial skills is shrinking, differences persist. Though small in scope, this was evident in the ANCOVA results for the Card Rotation Test of this study (see Table 2). Males and females responded differently to treatment. In particular, why did females experiencing treatment earn substantially lower adjusted mean scores? If a study was to focus on instructing females separately from males would the results differ? What type of spatial training would be most beneficial for both males and females?

A final need is to further explore the use of Origami in the mathematics classroom. It is clearly a supported instructional method, yet little research exists to validate the claims made by numerous authors. This study has begun this process showing that in some ways Origami can be a beneficial experience for students. Future studies "should use Origami in a variety of mathematical settings to test for other abilities that might be strengthened by this art" (Boakes, 2006, p. 136).

Conclusion

It was the intent of this article to reveal the potential that Origami-mathematics lessons have in the mathematics classroom. While this study is small in scale, it does contribute further evidence that Origami can be an effective instructional tool. From the teacher's perspective, consider what took place here. Three times a week, approximately 20 minutes was taken from traditional instruction to teach these Origami lessons. Though nearly one hour of contact time per week was taken from the experimental group's class sessions, they performed as well as their counterparts taught in a traditional manner on the geometry knowledge test. The same results were found for two of three spatial tests. Though the actual analysis did not reveal statistical significance, students did as well as with Origami blended into geometry instruction as they did without it. Thus, it does provide the teacher with the confidence that Origami lessons can contribute to geometry understanding and spatial skills.

Knowing that purely quantitative work may not show the true effect an instructional strategy will have on learning, the researcher also collected some informal qualitative feedback through a questionnaire given to participants at the end of the study. Though not made a part of the formal study and analysis, the information shared may be useful for the practitioner who is unsure of the use of such a method. When students were asked to write down one word to describe their experiences, students responded in an overwhelmingly positive manner with words like "fun," "helpful," "enjoyable," and "awesome." How often do students respond like this to math instruction? (Only one individual responded with a negative statement.) Students were also asked to what extent the Origami lessons helped them understand geometry using a 1–5 Likert scale response. With 5 representing strong agreement and 1 strong disagreement, the mean response was 4.36. Thus, students felt as if the Origami lessons helped them. Though this instrument only captures the opinion and attitude of the middle school student, it should further support the conclusion that Origami can be a positive influence on the learning and understanding of geometric concepts. If nothing else, it provides the math teacher with a way to enhance traditional mathematics instruction.

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Appendix A

Sample Origami-Mathematics Lesson

Lesson #1—Sailboat Model

Materials needed:
1 square sheet of standard Origami paper

Math Concepts:
Shape, area, parallel and perpendicular, spatial relations

Math Vocabulary:
Parallel lines
Perpendicular lines
Angles—acute, obtuse, right
Right triangle
Quadrilateral
Trapezoid
Area

Key Questions:

1. When you make both folds what shapes do you make? *[Squares]* How do the areas of the new squares compare to the old ones? *[They are one-fourth of the original square.]* What about the fold lines, do you recognize them? *[Yes, they are perpendicular lines.]* How do you know? *[They meet at right angles.]*

2. When you fold in the diagonal of the square, what kind of shapes do you have now? *[Squares and right triangles.]* Where do all the fold lines meet? *[At the midpoint of the segments.]* What kind of angles can you find if you darken in the line segments? *[Have students show where acute, obtuse, and right angles are formed.]*

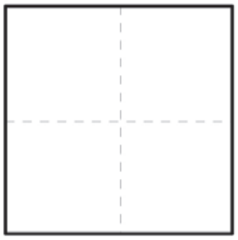
3. What kind of shapes do you see once you fold the corners in? *[Right triangles and squares again.]* Can you find parallel or perpendicular lines anywhere? *[Have students show where they are on the model.]*

4. Once you squash-fold your model, what shapes do you find? *[Right triangles.]* How does the area of the red triangle compare to the two smaller white ones? *[It is twice as big.]* Can you still find parallel or perpendicular line segments? *[Have students show where they are.]*

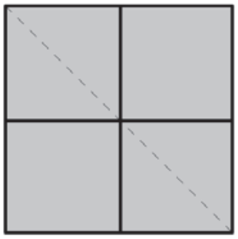
5. With the last fold done, what shape is the base of the boat? *[Quadrilateral.]*
   Does it have a special name? *[Trapezoid.]*

Sailboat—Folding instructions

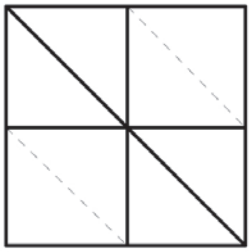
1. Start with white side up. Fold one side of square to meet its opposite side. Do this for both sets of opposite sides.



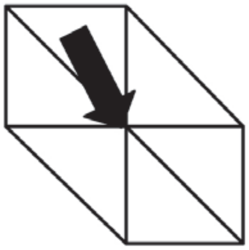
2. Flip paper over to colored side. Then fold one corner of the square to meet an opposite corner of a square. Crease along fold line then open back up to the original square.



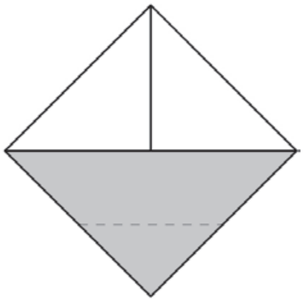
3. Flip back to the white side. Take one set of opposite corners and fold them into the center of the square, where the two existing fold lines meet.

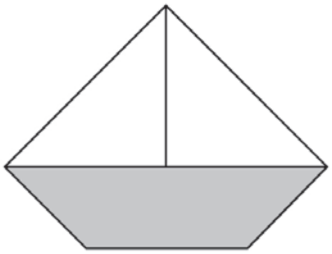


4. Hold paper in a cupped hand so it sits naturally in it. Gently push finger in center of square where folds all meet at a point. Carefully squash along folds to make shape shown.



5. Fold the colored corner up to meet the two right angles of the white triangles.





Source: *unknown*